

# On Gauge Dynamics and SUSY Breaking in Orientiworld

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## Abstract

In the Orientiworld framework the Standard Model fields are localized on D3-branes sitting on top of an orientifold 3-plane. The transverse 6-dimensional space is a non-compact orbifold (or a more general conifold). The 4-dimensional gravity on D3-branes is reproduced due to the 4-dimensional Einstein-Hilbert term induced at the quantum level. The orientifold 3-plane plays a crucial role, in particular, without it the D3-brane world-volume theories would be conformal due to the tadpole cancellation. We study non-perturbative gauge dynamics in various  $\mathcal{N} = 1$  supersymmetric orientiworld models based on the  $\mathbf{Z}_3$  as well as  $\mathbf{Z}_5$  and  $\mathbf{Z}_7$  orbifold groups. Our discussions illustrate that there is a rich variety of supersymmetry preserving dynamics in some of these models. On the other hand, we also find some models with dynamical supersymmetry breaking.

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# I. INTRODUCTION

Extra dimensions naturally arise in superstring theory (or M-theory), which is believed to be a consistent theory of quantum gravity. However, in order to model the real world with critical string theory (or M-theory), one must address the question of why the extra dimensions have not been observed. One way to make extra dimensions consistent with observation is to assume that they are compact with small enough volume. If the Standard Model gauge and matter fields propagate in such extra dimensions (as is the case in, say, weakly coupled heterotic string theory), then their linear sizes should not be larger than about inverse TeV [1]. On the other hand, in the Brane World scenario the Standard Model gauge and matter fields are assumed to be localized on branes (or an intersection thereof), while gravity lives in a larger dimensional bulk of space-time. Such a scenario with compact extra dimensions can, for instance, be embedded in superstring theory via Type I' compactifications. Then the extra dimensions transverse to the branes can have sizes as large as about a tenth of a millimeter [2].

To begin with considering compact (or, more generally, finite volume) extra dimensions was motivated by the requirement that at the distance scales for which gravity has been measured one should reproduce 4-dimensional gravity. However, as was pointed out in [3,4], 4-dimensional gravity can be reproduced even in theories with infinite-volume extra dimensions. In particular, according to [4] 4-dimensional gravity can be reproduced on a 3-brane in infinite-volume bulk (with 6 or more space-time dimensions) up to ultra-large distance scales. Thus, in these scenarios gravity is almost completely localized on a brane (which is almost  $\delta$ -function-like) with ultra-light modes penetrating into the bulk. As was explained in [4], this dramatic modification of gravity in higher codimension models with infinite volume extra dimensions is due to the Einstein-Hilbert term on the brane, which is induced via loops of non-conformal brane matter [3,4].

In [5] we described an explicit string theory framework for embedding models with infinite-volume extra dimensions. In this framework, which we refer to as Orientiworld, the Standard Model gauge and matter fields are localized on (a collection of) D3-branes embedded in infinite-volume extra space. In particular, we consider unoriented Type IIB backgrounds in the presence of some number of D3-branes as well as an orientifold 3-plane embedded in an orbifolded space-time. The D3-brane world-volume theory in this framework is non-conformal (at least for some backgrounds with at most  $\mathcal{N} = 1$  supersymmetry). At the quantum level we have the Einstein-Hilbert term induced on the branes, which leads to almost complete localization of gravity on the D3-branes. In particular, as was discussed in [5], (at least in some backgrounds) up to an ultra-large cross-over distance scale the gravitational interactions of the Standard Model fields localized on D3-branes are described by 4-dimensional laws of gravity.

The orientiworld framework appears has a rich structure for model building. In particular, since the extra dimensions have infinite volume, the number of D3-branes is arbitrary. Moreover, the number of allowed orbifold groups is infinite. Thus, *a priori* the orbifold group can be an arbitrary<sup>1</sup> subgroup of  $Spin(6)$ , or, if we require  $\mathcal{N} = 1$  supersymmetry to

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<sup>1</sup>More precisely, there is a mild restriction on allowed orbifold groups if we require modular

avoid bulk tachyons, of  $SU(3)$ . To obtain a finite string background, we still must impose *twisted* tadpole cancellation conditions. However, twisted tadpoles must also be canceled in compact Type IIB orientifolds. Then the number of consistent solutions of the latter type is rather limited [6] as we can only have a finite number of D3-branes, and, moreover, the number of allowed orbifold groups is also finite as they must act crystallographically on the compact space. On the other hand, as we already mentioned, in the orientiworld framework the number of consistent solutions is *infinite*, which is encouraging for phenomenologically oriented model building.

This richness of the orientiworld framework can be exploited to construct various models for phenomenological applications. Thus, in [5] we gave a construction of an  $\mathcal{N} = 1$  supersymmetric 3-generation Pati-Salam model in the orientiworld framework. The purpose of this paper is to explore non-perturbative gauge dynamics in orientiworld models. In particular, we discuss  $\mathcal{N} = 1$  orientiworld models based on  $\mathbf{Z}_3$  as well as  $\mathbf{Z}_5$  and  $\mathbf{Z}_7$  orbifolds. The examples we study illustrate that various non-perturbative phenomena can be expected in orientiworld models including dynamical supersymmetry breaking.

The rest of this paper is organized as follows. In section II we review the orientiworld framework. In section III we discuss various couplings in unoriented Type IIB backgrounds. In section IV we discuss the  $\mathbf{Z}_3$  models. In section V we give the  $\mathbf{Z}_5$  and  $\mathbf{Z}_7$  examples. We give some concluding remarks in section VI.

## II. ORIENTIWORLD FRAMEWORK

In this section we review the orientiworld framework. First we describe the underlying oriented Type IIB orbifold backgrounds. We then consider their orientifolds. Parts of our discussion here will closely [7,8,5].

### A. Oriented Backgrounds

Consider Type IIB string theory with  $N$  parallel coincident D3-branes where the space transverse to the D-branes is  $\mathcal{M} = \mathbf{R}^6/\Gamma$ . The orbifold group  $\Gamma = \{g_a \mid a = 1, \dots, |\Gamma|\}$  ( $g_1 = 1$ ) must be a finite discrete subgroup of  $Spin(6)$  (it can be a subgroup of  $Spin(6)$  and not  $SO(6)$  as we are dealing with a theory containing fermions). If  $\Gamma \subset SU(3)$  ( $SU(2)$ ), we have  $\mathcal{N} = 1$  ( $\mathcal{N} = 2$ ) unbroken supersymmetry, and  $\mathcal{N} = 0$ , otherwise.

Let us confine our attention to the cases where type IIB on  $\mathcal{M}$  is a modular invariant theory<sup>2</sup>. The action of the orbifold on the coordinates  $X_i$  ( $i = 1, \dots, 6$ ) on  $\mathcal{M}$  can be described in terms of  $SO(6)$  matrices:  $g_a : X_i \rightarrow \sum_j (g_a)_{ij} X_j$ . (The action of  $g_a$  on the world-sheet superpartners of  $X_i$  is the same.) We also need to specify the action of the

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invariance of the closed string sector in the corresponding oriented Type IIB background.

<sup>2</sup>This is always the case if  $\Gamma \subset SU(3)$ . For non-supersymmetric cases this is also true provided that  $\nexists \mathbf{Z}_2 \subset \Gamma$ . If  $\exists \mathbf{Z}_2 \subset \Gamma$ , then modular invariance requires that the set of points in  $\mathbf{R}^6$  fixed under the  $\mathbf{Z}_2$  twist has real dimension 2.

orbifold group on the Chan-Paton charges carried by the D3-branes. It is described by  $N \times N$  matrices  $\gamma_a$  that form a representation of  $\Gamma$ . Note that  $\gamma_1$  is an identity matrix and  $\text{Tr}(\gamma_1) = N$ .

The D-brane sector of the theory is described by an oriented open string theory. In particular, the world-sheet expansion corresponds to summing over oriented Riemann surfaces with arbitrary genus  $g$  and arbitrary number of boundaries  $b$ , where the boundaries of the world-sheet correspond to the D3-branes. In [7] it was shown that the one-loop massless (and, in non-supersymmetric cases, tachyonic) tadpole cancellation conditions require that

$$\text{Tr}(\gamma_a) = 0 \quad \forall a \neq 1. \quad (1)$$

In [7] it was also shown that this condition implies that the Chan-Paton matrices  $\gamma_a$  form an  $n$ -fold copy of the *regular* representation of  $\Gamma$ . The regular representation decomposes into a direct sum of all irreducible representations  $\mathbf{r}_i$  of  $\Gamma$  with degeneracy factors  $n_i = |\mathbf{r}_i|$ . The gauge group is ( $N_i \equiv nn_i$ )

$$G = \otimes_i U(N_i). \quad (2)$$

The matter consists of Weyl fermions and scalars transforming in bifundamentals  $(\mathbf{N}_i, \bar{\mathbf{N}}_j)$  (see [10] for details). The overall center-of-mass  $U(1)$ , which is inherited from the parent  $\mathcal{N} = 4$  supersymmetric  $U(N)$  gauge theory and is always present in such models, is free - matter fields are not charged under this  $U(1)$ . We do, however, have matter charged under the rest of the  $U(1)$ 's, which we will refer to as non-trivial  $U(1)$ 's. If  $\Gamma \subset SO(3)$ , then all the non-trivial  $U(1)$ 's are anomaly free, in fact, in these cases gauge theories are necessarily non-chiral. As was discussed in [5], these  $U(1)$ 's acquire masses at the one-loop level via couplings to the corresponding twisted R-R two-forms for which there are induced kinetic terms on D3-branes. If  $\Gamma \subset SU(3)$  but  $\Gamma \not\subset SO(3)$ , then we have chiral gauge theories and some of the non-trivial  $U(1)$  factors are actually anomalous (in particular, we have  $U(1)_k SU(N_l)^2$  mixed anomalies), and are broken (that is, acquire masses) at the tree-level via a generalized Green-Schwarz mechanism [11,12,5]. If in these cases we also have anomaly-free non-trivial  $U(1)$ 's, then the latter acquire masses at the one-loop level just as in the  $\Gamma \subset SO(3)$  cases [5]. So the non-trivial  $U(1)$  factors decouple in the infra-red. As to the non-Abelian parts of the gauge theories, it was shown in [7] that they are conformal in the large  $N$  limit<sup>3</sup>, including in the non-supersymmetric cases. The key reason for this conformal property is the tadpole cancellation condition (1), which, as was explained in [7], implies that all planar diagrams<sup>4</sup> with external lines corresponding to non-Abelian gauge as well as matter fields reduce to those of the parent  $\mathcal{N} = 4$  theory, which is conformal. At finite  $N$  conformality of the non-Abelian parts of the  $\mathcal{N} = 2$  gauge theories becomes evident from vanishing of the one-loop  $\beta$ -function (as  $\mathcal{N} = 2$  gauge theories perturbatively are not renormalized beyond one loop), and can also be argued for  $\mathcal{N} = 1$  cases [7]. In

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<sup>3</sup>In this limit we take the string coupling  $g_s \rightarrow 0$  together with  $N \rightarrow \infty$  while keeping  $Ng_s$  fixed.

<sup>4</sup>In the large  $N$  limit non-planar diagrams are suppressed by powers of  $1/N$ .

non-supersymmetric cases, however, we always have twisted closed string sectors tachyons, which prevent one from considering finite  $N$  cases<sup>5</sup>.

## B. Unoriented Backgrounds

Let us now consider a generalization of the above setup by including orientifold planes. In the following we will mostly be interested in finite  $N$  theories, so let us focus on theories with at least  $\mathcal{N} = 1$  unbroken supersymmetry. Thus, consider Type IIB string theory on  $\mathcal{M} = \mathbf{C}^3/\Gamma$  where  $\Gamma \subset SU(3)$ . Consider the  $\Omega(-1)^{F_L}J$  orientifold of this theory, where  $\Omega$  is the world-sheet parity reversal,  $F_L$  is the fermion number operator, and  $J$  is a  $\mathbf{Z}_2$  element ( $J^2 = 1$ ) acting on the complex coordinates  $z_i$  ( $i = 1, 2, 3$ ) on  $\mathbf{C}^3$  such that the set of points in  $\mathbf{C}^3$  fixed under the action of  $J$  has real dimension  $\Delta = 0$  or 4.

If  $\Delta = 0$  then we have an orientifold 3-plane. If  $\Gamma$  has a  $\mathbf{Z}_2$  subgroup, then we also have an orientifold 7-plane. If  $\Delta = 4$  then we have an orientifold 7-plane. We may also have an orientifold 3-plane depending on whether  $\Gamma$  has an appropriate  $\mathbf{Z}_2$  subgroup. Regardless of whether we have an orientifold 3-plane, we can *a priori* introduce an arbitrary number of D3-branes<sup>6</sup>. On the other hand, if we have an orientifold 7-plane we must introduce 8 of the corresponding D7-branes to cancel the corresponding R-R charge appropriately. (The number 8 of D7-branes is required by the corresponding untwisted tadpole cancellation conditions.)

We need to specify the action of  $\Gamma$  on the Chan-Paton factors corresponding to the D3- and D7-branes (if the latter are present, which is the case if we have an orientifold 7-plane). Just as in the previous subsection, these are given by Chan-Paton matrices which we collectively refer to as  $\gamma_a^\mu$ , where the superscript  $\mu$  refers to the corresponding D3- or D7-branes. Note that  $\text{Tr}(\gamma_1^\mu) = n^\mu$  where  $n^\mu$  is the number of D-branes labelled by  $\mu$ .

Now the world-sheet expansion contains oriented as well as unoriented Riemann surfaces. The unoriented Riemann surfaces contain handles and boundaries as well as cross-caps. The latter are the (coherent Type IIB) states that describe the familiar orientifold planes. The presence of the cross-caps modifies the twisted one-loop tadpole cancellation conditions, which can now be written as:

$$B_a + \sum_{\mu} C_a^\mu \text{Tr}(\gamma_a^\mu) = 0, \quad a \neq 1. \quad (3)$$

These should be contrasted with (1) in the oriented case. In particular, in certain cases some  $B_a$ , which correspond to contributions due to cross-caps, need not vanish. This makes possible (albeit does not guarantee - see below) non-conformal gauge theories on D3-branes in the orientiworld context [8,9,5].

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<sup>5</sup>In the large  $N$  limit the closed twisted sector tachyons are harmless as the string coupling  $g_s$  goes to zero. Also, in this limit all non-trivial  $U(1)$ 's decouple in the infra-red.

<sup>6</sup>In general, codimension-3 and higher objects (that is D-branes and orientifold planes) do not introduce untwisted tadpoles.

Thus, let us see what kind of orientifold models we can have. For definiteness let us focus on the cases where we do have an orientifold 3-plane (that is,  $\Delta = 0$ ). If there are no orientifold 7-planes (that is, if  $\Gamma$  does not contain a  $\mathbf{Z}_2$  element), then the orientifold projection  $\Omega$  can be either of the  $SO$  or the  $Sp$  type: the corresponding orientifold 3-plane is referred to as  $O3^-$  or  $O3^+$ , respectively. That is, before orbifolding, if we place  $2N$  D3-branes on top of the  $O3^-$ -plane ( $O3^+$ -plane), we have the  $\mathcal{N} = 4$  super-Yang-Mills theory with the  $SO(2N)$  ( $Sp(2N)$ ) gauge group<sup>7</sup>. (We are using the convention where  $Sp(2N)$  has rank  $N$ .) After the orbifold projections the 33 (that is, the D3-brane) gauge group is a subgroup of  $SO(2N)$  ( $Sp(2N)$ ), which can contain  $U(N_k)$  factors as well as  $SO$  ( $Sp$ ) subgroups. The 33 matter can contain bifundamentals in any of these subgroups as well as rank-2 antisymmetric (symmetric) representations in the unitary subgroups. Next, if we have an  $O7$ -plane, the orientifold projection  $\Omega$  must always be of the  $SO$  type on the D7-branes - this is required by the tadpole cancellation condition. This, in particular, implies that the 33 and 77 matter cannot contain rank-2 symmetric representations. Note that we also have 37 matter in bifundamentals of the 33 and 77 gauge groups. Finally, note that there is no overall center-of-mass  $U(1)$  in these models (the parent theory is an  $SO$  or  $Sp$  gauge theory), so all  $U(1)$ 's (if present) are non-trivial in the sense of the previous subsection.

If  $\Gamma \subset SO(3)$ , then the corresponding gauge theories are necessarily non-chiral. If we have  $U(1)$  factors, they acquire masses at the one-loop level via the mechanism discussed in the previous subsection [5]. As to the non-Abelian parts of the corresponding gauge theories, as was discussed in [5], they are always conformal despite the fact that some twisted  $B_a$  can be non-zero in such models (see [5] for a detailed explanation of why this is so)<sup>8</sup>. The situation is very different in the cases where  $\Gamma \subset SU(3)$  but  $\Gamma \not\subset SO(3)$ . All such theories are non-conformal [5]. In fact, generically they are chiral with a few (essentially trivial) exceptions. Thus, in some cases the twisted tadpole cancellation conditions allow a choice such that all twisted Chan-Paton matrices are trivial (that is, they are identity matrices). In such a case the gauge theory is a pure  $SO$  or  $Sp$   $\mathcal{N} = 1$  super-Yang-Mills theory (that is, we have no chiral matter supermultiplets). In all other cases we have chiral matter. More precisely, there is one possible exception where the gauge group is  $SU(4) \otimes U(1)_A$  with matter transforming in  $\mathbf{6}(+2)$  (the  $U(1)_A$  charge is given in parenthesis). Such a theory is conformal as the matter is non-trivially charged under the anomalous  $U(1)_A$ , but the latter is broken at the tree level via a generalized Green-Schwarz mechanism, and the resulting non-Abelian gauge theory turns out to be non-conformal (as  $\mathbf{6}$  of  $SU(4)$  is a real representation). In general, if we have chiral matter, we have at least one anomalous  $U(1)$ . Such anomalous  $U(1)$ 's acquire masses at the tree level [11,12,5]. If in these cases we also have anomaly-free  $U(1)$ 's, they acquire masses at the one-loop level as in the  $\Gamma \subset SO(3)$  cases [5].

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<sup>7</sup>Note that we can also place  $2N + 1$  D3-branes on top of the  $O3^-$ -plane to obtain the  $SO(2N + 1)$  gauge group.

<sup>8</sup>The  $\Gamma \subset SU(2)$  orientifolds were originally discussed in [8,13,14].

### III. VARIOUS COUPLINGS IN ORIENTIWORLD

For our subsequent discussions it will be useful to understand some couplings in the orientifold backgrounds. For simplicity we will focus on the cases with O3-planes but without O7-planes. In fact, we will specialize on orbifold groups  $\Gamma = \mathbf{Z}_p$  such that  $\Gamma \subset SU(3)$  but  $\Gamma \not\subset SO(3)$ , where  $p$  is a prime. The action of the generator  $\theta$  or  $\mathbf{Z}_p$  on the complex coordinates  $z_\alpha$ ,  $\alpha = 1, 2, 3$ , on  $\mathbf{C}^3/\Gamma$  is given by

$$\theta z_\alpha = \omega^{\ell_\alpha} z_\alpha , \quad (4)$$

where  $\omega \equiv \exp(2\pi i/p)$ , and  $\sum_{\alpha=1}^3 \ell_\alpha = p$ .

#### A. Oriented Backgrounds

Before we turn to the unoriented backgrounds, let us recall some facts about the oriented theories. Thus, consider Type IIB on  $\mathbf{R}^{1,3} \times (\mathbf{C}^3/\mathbf{Z}_p)$ . The closed string sector has  $p-1$  twisted sectors  $\theta^k$ ,  $k = 1, \dots, p-1$ . In each twisted sector we have a complex NS-NS scalar  $\phi_k$  and a complex R-R two-form  $C_k$ , which satisfy the reality condition [15]

$$\phi_{p-k} = \phi_k^* , \quad (5)$$

and similarly for  $C_k$ .

Next, consider  $N$  D3-branes placed at the orbifold fixed point in  $(\mathbf{C}^3/\mathbf{Z}_p)$ . Let  $\mathcal{F}$  be the  $(N \times N$  matrix valued) D3-brane gauge field strength, which satisfies the orbifold projection  $\gamma_k \mathcal{F} \gamma_k^{-1} = \mathcal{F}$  (recall that we have  $N = np$  and  $\gamma_k = \text{diag}(I_n, \omega I_n, \omega^2 I_n, \dots, \omega^{p-1} I_n)$ , where  $I_n$  is the  $n \times n$  identity matrix). We have a Chern-Simons coupling of the following form [16,17,19]:

$$S_{\text{CS}} = \frac{1}{2\pi\alpha'} \sum_{k=1}^{p-1} \int_{\text{D3}} C_k \wedge \text{Tr} \left( \gamma_k e^{2\pi\alpha' \mathcal{F}} \right) . \quad (6)$$

In particular, the term linear in  $\mathcal{F}$

$$\sum_{k=1}^{p-1} \int_{\text{D3}} C_k \wedge \text{Tr} (\gamma_k \mathcal{F}) \quad (7)$$

describes the mixing between the  $p-1$  twisted two-forms  $C_k$  and  $p-1$  anomalous  $U(1)$ 's (note that we have  $p$   $U(1)$ 's, but one of them is an overall center-of-mass  $U(1)$  which does not couple to the twisted two-forms  $C_k$ ). Since the fields  $C_k$  have non-vanishing kinetic terms supported at the orbifold fixed point (that is, they propagate in  $\mathbf{R}^{1,3}$  that coincides with the D3-brane world-volumes), the anomalous  $U(1)$ 's are actually massive already at the tree level.

Now consider the supersymmetric completion of the couplings (7), which gives the corresponding Fayet-Iliopoulos (FI) couplings:

$$S_{\text{FI}} = \sum_{k=1}^{p-1} \int_{\text{D3}} \phi_k \text{Tr} (\gamma_k \mathcal{D}) , \quad (8)$$

where  $\mathcal{D}$  is the (matrix valued) auxiliary field corresponding to  $\mathcal{F}$ . The FI term for a given non-trivial  $U(1)_j$ ,  $j = 1, \dots, p-1$ , therefore reads:

$$\xi_{\text{FI},j} = \sum_{k=1}^{p-1} \phi_k \text{Tr}(\gamma_k \lambda_j) , \quad (9)$$

where  $\lambda_j$  is the Chan-Paton matrix corresponding to this  $U(1)_j$ . Note that at the orbifold point the D-terms give masses to the twisted NS-NS scalars  $\phi_k$ , which are now part of the massive  $U(1)$  gauge supermultiplets.

Next, consider the terms in (6) quadratic in  $\mathcal{F}$ :

$$\pi\alpha' \sum_{k=1}^{p-1} \int_{\text{D3}} C_k \wedge \text{Tr}(\gamma_k \mathcal{F}^2) . \quad (10)$$

The supersymmetric completion of this coupling gives a coupling proportional to

$$\sum_{k=1}^{p-1} \int_{\text{D3}} \phi_k \text{Tr}(\gamma_k \mathcal{F}^2) . \quad (11)$$

That is, the twisted NS-NS scalars contribute to the gauge couplings, while the twisted R-R scalars (dual to the twisted two-forms) contribute to the corresponding  $\theta$ -angles (that is, the gauge kinetic function is given by  $f = S + f_1$ , where  $S$  is the untwisted sector dilaton supermultiplet, while  $f_1$  is the contribution which depends on the twisted closed string sector moduli) [10,11].

## B. Unoriented Backgrounds

Once we add an O3-plane, the above discussion is modified as follows. First, note that the twisted Chan-Paton matrices now have the form:  $\gamma_k = \text{diag}(I_{n_0}, \omega I_{n_1}, \dots, \omega^{p-1} I_{n_{p-1}})$ , where  $\sum_{k=0}^{p-1} n_k = N$ , and the integers  $n_k$  satisfy the reality condition  $n_{p-k} = n_k$ ,  $k = 1, \dots, p-1$ , and otherwise are no longer identical but are determined by the twisted tadpole cancellation conditions [18,8,9,11] (here  $\eta = -1$  for the O3<sup>-</sup> plane, while  $\eta = 1$  for the O3<sup>+</sup>-plane):

$$\text{Tr}(\gamma_{2k}) = -4\eta \prod_{\alpha=1}^3 (1 + \omega^{k\ell_\alpha})^{-1} . \quad (12)$$

In particular, some  $n_k$  can actually vanish. The gauge group is now  $SO(n_0)$  or  $Sp(n_0)$  (depending on whether we choose an O3<sup>-</sup> or O3<sup>+</sup>-plane) times  $\bigotimes_{k=1}^{(p-1)/2} U(n_k)$  (if any of the  $n_k$  vanishes, we simply delete the corresponding subgroup). Second, the orientifold projection removes the real parts of the complex fields  $\phi_k$  and  $C_k$ , while the imaginary parts  $\text{Im}(\phi_k)$  and  $\text{Im}(C_k)$ ,  $k = 1, \dots, (p-1)/2$ , are combined (after dualizing the two-forms  $C_k$  to scalars  $\tilde{\phi}_k$ ) into  $(p-1)/2$  twisted chiral supermultiplets [19,20]. If we actually have an anomalous  $U(1)_{n_j}$  factor coming from the  $U(n_j)$  subgroup (that is, if the corresponding  $n_j \neq 0$ ), then this  $U(1)_{n_j}$  becomes massive at the tree level via the Chern-Simons coupling of the corresponding field strength  $\mathcal{F}_{n_j}$  with the fields  $\text{Im}(C_k)$ . This can be seen from the part of (7) surviving the orientifold projection (it is not difficult to see that the Chan-Paton



matrix  $\lambda_{n_j}$  for this  $U(1)_{n_j}$  factor is a matrix block-diagonal w.r.t. partitions of  $N$  into  $n_0, \dots, n_{p-1}$  integers with only non-vanishing entries being  $n_j$  1's and  $n_{p-j}(=n_j)$   $-1$ 's):

$$-2 \sum_{k=1}^{(p-1)/2} \sum_{j=1}^{(p-1)/2} \text{Im} \left( \text{Tr} \left[ \gamma_k \lambda_{n_j} \right] \right) \int_{\text{D}3} \text{Im}(C_k) \wedge \mathcal{F}_{n_j} . \quad (13)$$

Similarly, the Fayet-Iliopoulos terms are given by:

$$\xi_{\text{FI}, n_j} = -2 \sum_{k=1}^{(p-1)/2} \text{Im}(\phi_k) \text{Im} \left( \text{Tr} \left[ \gamma_k \lambda_{n_j} \right] \right) . \quad (14)$$

These couplings imply that, since we have  $m_A = \sum_{j=1}^{(p-1)/2} [1 - \delta_{n_j, 0}]$  anomalous  $U(1)$ 's, precisely  $m_A$  linear combinations of the  $(p-1)/2$  chiral superfields (whose lowest components are given by complex scalars  $\Phi_k = \text{Im}(\phi_k) + i \text{Im}(\tilde{\phi}_k)$ ) become part of the massive  $U(1)$  superfields at the orbifold point.

Finally, let us see what happens to the correction  $f_1$  to the gauge kinetic function due to the twisted moduli  $\Phi_k$ . In the unoriented backgrounds these corrections actually *vanish* [20]. Thus, consider the part of (11) surviving the orientifold projection. For the  $SO(n_0)/Sp(n_0)$  part of the gauge group (if present) the corresponding part of the trace  $\text{Tr}(\gamma_k \mathcal{F}^2)$  is always real, so due to the reality condition  $\phi_{p-k} = \phi_k^*$  only the real parts of  $\phi_k$  can contribute. But it is precisely the real parts of  $\phi_k$  that are removed by the orientifold projection. As to the  $U(n_j)$  subgroups, note that up to the appropriate normalization factors the corresponding corrections to  $f_1$  are the same for the non-Abelian parts  $SU(n_j)$  as those for the Abelian parts  $U(1)_{n_j}$ . The latter are given by

$$\sum_{k=1}^{p-1} \sum_{j=1}^{(p-1)/2} \text{Tr} \left( \gamma_k \lambda_{n_j}^2 \right) \int_{\text{D}3} \phi_k \mathcal{F}_{n_j}^2 . \quad (15)$$

Note that the traces  $\text{Tr}(\gamma_k \lambda_{n_j}^2)$  are all real, so only the real parts of  $\phi_k$  can contribute. This implies that in the orientifold backgrounds the gauge couplings (and the corresponding  $\theta$ -angles) do not receive twisted moduli dependent corrections at the tree level [20].

#### IV. THE $\mathbf{Z}_3$ MODELS

The simplest choice of the orbifold group  $\Gamma$  in the present context is  $\Gamma = \mathbf{Z}_3 = \{1, \theta, \theta^2\}$ , where the generator  $\theta$  of  $\mathbf{Z}_3$  acts on the complex coordinates  $z_\alpha$ ,  $\alpha = 1, 2, 3$ , on  $\mathbf{C}^3$  as follows:

$$\theta z_\alpha = \omega z_\alpha , \quad (16)$$

where  $\omega \equiv \exp(2\pi i/3)$ . At the origin of  $\mathbf{C}^3/\Gamma$  we can place an  $\text{O}3^-$ - or  $\text{O}3^+$ -plane. Let  $\eta = -1$  in the former case, while  $\eta = +1$  in the latter case. The twisted tadpole cancellation requires that [8]

$$\text{Tr}(\gamma_\theta) = 4\eta . \quad (17)$$

We can then place  $(3N + 4\eta)$  D3-branes on top of the  $\text{O}3$ -plane, and choose

$$\gamma_\theta = \text{diag}(\omega I_N, \omega^{-1} I_N, I_{N+4\eta}) . \quad (18)$$

The twisted closed string sector gives rise to a single massless chiral supermultiplet corresponding to the orbifold blow-up mode. The massless open string spectrum gives rise to the gauge theory on the D3-branes. For  $\eta = 1$  and  $N = 0$  we have pure  $Sp(4)$  super-Yang-Mills theory<sup>9</sup>. For  $\eta = 1$  and  $N \in 2\mathbf{N}$  we have  $SU(N) \otimes Sp(N+4) \otimes U(1)_A$  gauge theory with matter in the chiral supermultiplets  $\Phi_\alpha = 3 \times (\mathbf{S}, \mathbf{1})(+2)$  and  $Q_\alpha = 3 \times (\overline{\mathbf{N}}, \mathbf{N} + \mathbf{4})(-1)$ , where the anomalous  $U(1)_A$  charges are given in parentheses, and  $\mathbf{S}$  stands for the two-index  $N(N+1)/2$  dimensional symmetric representation of  $SU(N)$ . For  $\eta = -1$  and  $N = 4$  we have  $SU(4) \otimes U(1)_A$  gauge theory with matter in the chiral supermultiplets  $\Phi_\alpha = 3 \times \mathbf{6}(+2)$ . For  $\eta = -1$  and  $N = 5$  we have  $SU(5) \otimes U(1)_A$  gauge theory with matter in the chiral supermultiplets  $\Phi_\alpha = 3 \times \mathbf{10}(+2)$  and  $Q_\alpha = 3 \times \overline{\mathbf{5}}(-1)$ . For  $\eta = -1$  and  $N \geq 6$  we have  $SU(N) \otimes SO(N-4) \otimes U(1)_A$  gauge theory with matter in the chiral supermultiplets  $\Phi_\alpha = 3 \times (\mathbf{A}, \mathbf{1})(+2)$  and  $Q_\alpha = 3 \times (\overline{\mathbf{N}}, \mathbf{N} - \mathbf{4})(-1)$ , where  $\mathbf{A}$  stands for the two-index  $N(N-1)/2$  dimensional antisymmetric representation of  $SU(N)$ . (Note that in the  $\eta = -1$  and  $N = 6$  case the  $SO$  part of the gauge group is actually Abelian.) In the cases where we have the  $Q_\alpha$  matter, we have the following tree-level superpotential:

$$\mathcal{W}_{\text{tree}} = y \epsilon_{\alpha\beta\gamma} \Phi_\alpha Q_\beta Q_\gamma + \dots , \quad (19)$$

where  $y$  is the corresponding Yukawa coupling, and the ellipses stand for non-renormalizable couplings. Note that at the renormalizable level we have an  $SO(3)$  global symmetry<sup>10</sup>, and the subscript  $\alpha$  in  $\Phi_\alpha$  and  $Q_\alpha$  corresponds to the triplet of  $SO(3)$ .

### A. The $Sp$ Theories

Some examples (more precisely, their compact versions) of the above theories with  $\eta = -1$  (that is, with the  $SO$  orientifold projection) were discussed in [21,18,22,23,20]. Here we will focus on the theories with  $\eta = 1$  (that is, with the  $Sp$  orientifold projection). Let us note that the  $\eta = 1$  models are actually simpler to discuss than their  $\eta = -1$  counterparts. This is because in the  $\eta = -1$  cases the  $SU$  part of the gauge theory is asymptotically free (while the  $SO$  part is not asymptotically free), so in the infra-red we have to deal with a strongly coupled chiral gauge theory<sup>11</sup>. In contrast, in the  $\eta = 1$  case it is the  $SU$  part that is not asymptotically free, while the  $Sp$  part is, so in the infra-red the non-perturbative dynamics reduces to that of a strongly coupled  $Sp(N_c = N+4)$  gauge theory with  $N_f = 3N/2$  flavors

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<sup>9</sup>Recall that in our conventions  $Sp(N)$  with  $N \in 2\mathbf{N}$  has rank  $N/2$ .

<sup>10</sup>Non-renormalizable couplings suppressed by powers of  $M_s$  break this global  $SO(3)$  symmetry to its discrete subgroup subsumed in the discrete  $\mathbf{Z}_3$  gauge symmetry.

<sup>11</sup>One exception is the  $\eta = -1$  and  $N = 4$  model, where the non-Abelian gauge group is  $SU(4)$ , the matter consists of 3 chiral supermultiplets in  $\mathbf{6}$  of  $SU(4)$ , and there is no tree-level superpotential. This theory can be thought of as  $SO(6)$  with 3 flavors, which is well understood [24].

(by one flavor of  $Sp(N_c)$  we mean a pair of chiral supermultiplets in  $N_c$  of  $Sp(N_c)$ ). And these theories are well understood [25].

First, consider the model with  $\eta = 1$  and  $N = 0$ . The gauge theory on the D3-branes is pure  $Sp(4)$  super-Yang-Mills theory. Note that the twisted closed string sector gives rise to a single chiral supermultiplet, which plays no role in the gauge dynamics (this follows from our discussion in the previous section). Non-perturbatively we have gaugino condensate in  $Sp(4)$  with confinement and chiral symmetry breaking. This model is interesting as it provides a simple setup where one gets pure  $\mathcal{N} = 1$  superglue on D3-branes. In particular, it would be interesting to use this setup to identify BPS domain walls in the  $Sp(4)$  super-Yang-Mills theory, but this is outside of the scope of this paper.

Let us now discuss the  $\eta = 1$  and  $N \in 2\mathbb{N}$  models. For presentation purposes we will discuss the  $N = 2$  model after we discuss all the other cases.

### *The $SU(4) \otimes Sp(8) \otimes U(1)_A$ Model*

This is the  $\eta = 1$  and  $N = 4$  model. The gauge group is  $SU(4) \otimes Sp(8) \otimes U(1)_A$ , the chiral matter is given by  $\Phi_\alpha = 3 \times (\mathbf{10}, \mathbf{1})(+2)$  and  $Q_\alpha = 3 \times (\overline{\mathbf{4}}, \mathbf{8})(-1)$ , and the tree-level superpotential is given by (19). As we have already mentioned, the  $SU(4)$  gauge coupling is weak in the infra-red, while the  $Sp(8)$  gauge coupling becomes strong. The low energy degrees of freedom are given by mesons (note that there are no baryons in  $Sp$  theories [25])

$$\mathcal{M}_{[\alpha\beta]} = 3 \times (\overline{\mathbf{10}}, \mathbf{1})(-2) , \quad (20)$$

$$\mathcal{M}_{\{\alpha\beta\}} = 6 \times (\overline{\mathbf{6}}, \mathbf{1})(-2) . \quad (21)$$

(Note that for  $SU(4)$  we actually have  $\overline{\mathbf{6}} = \mathbf{6}$ .) There is no non-perturbative superpotential in this theory, and at the origin of the meson moduli space we have confinement without chiral symmetry breaking [25]. Note that due to the tree-level superpotential (19) the mesons  $\mathcal{M}_{[\alpha\beta]}$  pair up with the fields  $\Phi_\alpha$  and acquire masses:

$$\mathcal{W}_{\text{tree}} = y \epsilon_{\alpha\beta\gamma} \Phi_\alpha \mathcal{M}_{[\beta\gamma]} . \quad (22)$$

So at low energies we have the  $SU(4)$  gauge theory with matter chiral supermultiplets in  $P_I = 6 \times \overline{\mathbf{6}}(-2)$ ,  $I = 1, \dots, 6$ . Actually, so far we have been ignoring the anomalous  $U(1)_A$ . The corresponding D-term is given by

$$D = -2P^2 + \xi_{\text{FI}} . \quad (23)$$

From (14) it follows that  $\xi_{\text{FI}}$  is negative for positive values of  $\text{Im}(\phi_1)$  (which corresponds to the size of the blow-up). This implies that all the fields  $P_I$  as well as  $\text{Im}(\phi_1)$  have vanishing expectation values. That is, the blow-up mode is frozen and the orbifold *cannot* be blown up in this model. As we discussed in the previous section, at the orbifold point the twisted chiral superfield becomes part of the massive  $U(1)_A$  gauge supermultiplet. The low energy theory is therefore  $SO(6)$  gauge theory with 6 vectors. Note that this theory is not infra-red free. However, it has a dual magnetic description [24] in terms of  $SO(4) \sim SU(2)_L \otimes SU(2)_R$  gauge theory with 6 flavors of quarks  $q^I$  ( $I = 1, \dots, 6$ ) in the  $4 \sim (2, 2)$  dimensional representation along with  $6(6+1)/2 = 21$  gauge singlets  $M_{IJ}$  and the superpotential

$$\mathcal{W}_{\text{magnetic}} = M_{IJ} q^I q^J . \quad (24)$$

Note that the magnetic theory is actually free in the infra-red.

Before we discuss the  $\eta = 1$  and  $N = 6$  model, we would like to discuss the  $\eta = 1$  and  $N > 6$  models. In this case the  $Sp(N+4)$  theory has a dual magnetic description (albeit the magnetic theory is also strongly coupled) [25]. In this dual picture the gauge group is  $SU(N) \otimes Sp(2N-8) \otimes U(1)_A$ , the matter is given by

$$\Phi_\alpha = 3 \times (\mathbf{S}, \mathbf{1})(+2) , \quad (25)$$

$$\mathcal{M}_{[\alpha\beta]} = 3 \times (\overline{\mathbf{S}}, \mathbf{1})(-2) , \quad (26)$$

$$\mathcal{M}_{\{\alpha\beta\}} = 6 \times (\overline{\mathbf{A}}, \mathbf{1})(-2) , \quad (27)$$

$$q_\alpha = 3 \times (\mathbf{N}, \mathbf{2N}-\mathbf{8})(+1) , \quad (28)$$

and the superpotential is given by

$$\mathcal{W}_{\text{magnetic}} = y \epsilon_{\alpha\beta\gamma} \Phi_\alpha \mathcal{M}_{[\beta\gamma]} + \mathcal{M}_{\{\alpha\beta\}} q_\alpha q_\beta . \quad (29)$$

The mesons  $\mathcal{M}_{[\alpha\beta]}$  pair up with the fields  $\Phi_\alpha$  and acquire masses, so at low energies we have the mesons  $\mathcal{M}_{\{\alpha\beta\}}$  and the quarks  $q_\alpha$ . As we have already mentioned, the  $Sp(2N-8)$  gauge coupling is strong in the infra-red. On the other hand, the  $SU(N)$  gauge coupling remains weak (even after integrating out the fields  $\mathcal{M}_{[\alpha\beta]}$  and  $\Phi_\alpha$ ). Moreover, the orbifold blow-up mode is frozen in this model. This can be seen as follows. The  $U(1)_A$  D-term reads:

$$D = -2M_{\{\alpha\beta\}}^2 + q^2 + \xi_{\text{FI}} , \quad (30)$$

where  $\xi_{\text{FI}} < 0$  for positive  $\text{Im}(\phi_1)$ . We must also ensure D-flatness for the  $SU$  and  $Sp$  subgroups. The D-flat directions are in one-to-one correspondence with chiral gauge invariant operators. As far as the  $Sp$  part is concerned, such operators can only contain the following combinations:

$$\Sigma_{[\alpha\beta]} = q_{[\alpha} q_{\beta]} = 3 \times (\mathbf{S}, \mathbf{1})(+2) , \quad (31)$$

$$\Sigma_{\{\alpha\beta\}} = q_{\{\alpha} q_{\beta\}} = 6 \times (\mathbf{A}, \mathbf{1})(+2) . \quad (32)$$

Note, however, that  $\Sigma_{[\alpha\beta]}$  cannot enter as we cannot construct  $SU(N)$  gauge invariant operators from  $\Sigma_{[\alpha\beta]}$ ,  $\Sigma_{\{\alpha\beta\}}$  and  $M_{\{\alpha\beta\}}$ . On the other hand, the F-flatness conditions imply that  $\Sigma_{\{\alpha\beta\}} = 0$ . It then follows that the FI term must vanish along with the vacuum expectation values of  $\mathcal{M}_{\{\alpha\beta\}}$  and  $q_\alpha$ .

Let us now discuss the  $N = 6$  case. The above discussion is essentially unmodified in this case except that in the dual magnetic theory the  $Sp(4)$  subgroup is actually weakly coupled. As to the  $SU(6)$  gauge theory, its one-loop  $\beta$ -function coefficient vanishes (after integrating out the the fields  $\mathcal{M}_{[\alpha\beta]}$  and  $\Phi_\alpha$ ), but it is still free in the infra-red.

### The $SU(2) \otimes Sp(6) \otimes U(1)_A$ Model

This is the  $\eta = 1$  and  $N = 2$  model. This model is interesting as supersymmetry is dynamically broken in this model. The gauge group is  $SU(2) \otimes Sp(6) \otimes U(1)_A$ , the chiral matter is given by  $\Phi_\alpha = 3 \times (\mathbf{3}, \mathbf{1})(+2)$  and  $Q_\alpha = 3 \times (\mathbf{2}, \mathbf{6})(-1)$ , and the tree-level superpotential is given by (19). The  $U(1)_A$  D-term is given by

$$D = 2\Phi^2 - Q^2 + \xi_{\text{FI}} , \quad (33)$$

where  $\xi_{\text{FI}}$  is negative for positive values of  $\text{Im}(\phi_1)$ .

First consider the case where  $\Phi_\alpha = 0$ . Then the above D-term vanishes only if  $Q_\alpha = 0$  and  $\text{Im}(\phi_1) = 0$ . The  $SU(2)$  gauge coupling is weak in the infra-red, so as far as the  $Sp$  part of the gauge theory is concerned we have  $Sp(N_c = 6)$  with  $N_f = 3$  flavors. In general, the  $Sp(N_c)$  theory with  $N_f \leq N_c/2$  flavors (that is,  $2N_f$  fields  $Q_i$ ,  $i = 1, \dots, 2N_f$ , in  $N_c$  of  $Sp(N_c)$ ) has a dynamically generated superpotential [25]:

$$\mathcal{W}_{\text{non-pert}} \sim \left( \frac{\Lambda^{3(N_c/2+1)-N_f}}{\text{Pf}(M)} \right)^{1/(N_c/2+1-N_f)} , \quad (34)$$

where  $M_{ij} = -M_{ji} = Q_i Q_j$  are the meson fields, and  $\Lambda$  is the dynamically generated scale. For  $N_f = N_c/2$  the gauge group is completely broken for  $\text{Pf}(M) \neq 0$ , and this superpotential is generated by an instanton in the broken  $Sp(N_c)$ . For  $N_f < N_c/2$  the superpotential is associated with gaugino condensation in the unbroken  $Sp(N_c - 2N_f)$ . In particular, for  $0 < N_f \leq N_c/2$  the above superpotential has a runaway behavior w.r.t. the vacuum expectation values of  $M_{ij}$  (that is, there is no supersymmetric vacuum for finite values of  $M_{ij}$ ).

In our case the mesons are  $\mathcal{M}_{[\alpha\beta]} = 3 \times (\mathbf{3}, \mathbf{1})(-2)$  and  $\mathcal{M}_{\{\alpha\beta\}} = 6 \times (\mathbf{1}, \mathbf{1})(-2)$ . At first it might seem that we have global supersymmetry breaking as the  $U(1)_A$  D-term grows with non-vanishing vacuum expectation values of the mesons. However, recall that we have assumed that  $\Phi_\alpha = 0$ . But there is nothing stopping  $\Phi_\alpha$  from being non-vanishing. Suppose some or all  $\Phi_\alpha \neq 0$ . Then the D-term can *a priori* be set to zero. Nonetheless, we still have no supersymmetric vacuum. Thus, in this case due to the tree-level superpotential (19) two of the fields  $Q_\alpha$  acquire masses (this is independent of a particular configuration of the vacuum expectation values of  $\Phi_\alpha$  as long as at least one of the fields  $\Phi_\alpha \neq 0$ ). So as far as the  $Sp(6)$  part of the gauge theory is concerned, at low energies we have  $Sp(6)$  with one flavor. As we already mentioned above, in this theory we have a dynamically generated runaway (in the corresponding meson field) superpotential with no supersymmetric vacuum. The  $D$ -term, however, can now vanish, so there is no stable vacuum with broken global supersymmetry.

Even so, as was discussed in detail in [26], generically we do expect to have a stable vacuum with broken *local* supersymmetry. In particular, if in the context of global supersymmetry we have a runaway superpotential with the runaway directions corresponding to charged matter fields, then in the context of supergravity the runaway directions are generically expected to be stabilized due to contributions coming from the Kähler potential (see [26] for details). Note that this mechanism is four-dimensional, and we indeed have four-dimensional supergravity on D3-branes via the mechanism of [4].

## B. Comments on the $\eta = -1$ Models

In this subsection we would like to comment on some properties of the  $\mathbf{Z}_3$  models with  $\eta = -1$ , that is, those where we have an  $O3^-$ -plane. As we have already mentioned, compact versions of some of these models were discussed in [21,18,22,23,20].

### *The $SU(4) \otimes U(1)_A$ Model*

This is the  $\eta = -1$  and  $N = 4$  model. The gauge group is  $SU(4) \otimes U(1)_A$ , the chiral matter is given by  $\Phi_\alpha = 3 \times \mathbf{6}(+2)$ , and there is no tree-level superpotential. The  $U(1)_A$  D-term is given by

$$D = 2\Phi^2 + \xi_{\text{FI}} , \quad (35)$$

where  $\xi_{\text{FI}}$  is negative for positive values of  $\text{Im}(\phi_1)$ .

The  $SU(4)$  gauge theory is strongly coupled in the infra-red. We can view this theory as  $SO(6)$  with 3 vectors. In this theory there are two distinct branches [24]. The first branch has a dynamically generated runaway superpotential in the vacuum expectation values of the mesons  $\mathcal{M}_{\{\alpha\beta\}} = 6 \times \mathbf{1}(+2)$ . On this branch supersymmetry is broken via the mechanism mentioned at the end of the previous subsection. The second branch has a vanishing superpotential, so supersymmetry is intact. Note that in this model the orbifold blow-up mode can be non-zero. In fact, on the first branch the non-supersymmetric vacuum has a non-zero blow-up mode, while on the second branch it depends on the mesons  $\mathcal{M}_{\{\alpha\beta\}}$  (in both cases the blow-up mode is fixed from the requirement that the  $U(1)_A$  D-term vanish).

### *The $SU(5) \otimes U(1)_A$ Model*

This is the  $\eta = -1$  and  $N = 5$  model. The gauge group is  $SU(5) \otimes U(1)_A$ , the chiral matter is given by  $\Phi_\alpha = 3 \times \mathbf{10}(+2)$  and  $Q_\alpha = 3 \times \mathbf{\bar{5}}(-1)$ , and the tree-level superpotential is given by (19). The  $U(1)_A$  D-term in this model is given by

$$D = 2\Phi^2 - Q^2 + \xi_{\text{FI}} , \quad (36)$$

where  $\xi_{\text{FI}}$  is negative for positive values of  $\text{Im}(\phi_1)$ .

To analyze the gauge dynamics in this model, let us first consider the model with the gauge group  $SU(5)$ , chiral matter in  $\Phi_\alpha = 3 \times \mathbf{15}$  and  $Q_\alpha = 3 \times \mathbf{\bar{5}}$ , and *no* tree-level superpotential. This theory is *s*-confining [27]<sup>12</sup>. A simple way of understanding this is as follows. Consider the following gauge invariant operator:

$$\Sigma_{\{\alpha\beta\}} \equiv \Phi_{\{\alpha} \Phi_{\beta\}} \epsilon_{\gamma\delta\eta} \Phi_\gamma \Phi_\delta \Phi_\eta = 6 \times \mathbf{1} . \quad (37)$$

So we have a D-flat direction corresponding to turning on non-vanishing vacuum expectation values of  $\Phi_\alpha$ . The original  $SU(5)$  gauge group can be broken down to  $SU(2)$  along this direction. To see this, consider the branching of  $\mathbf{10}$  of  $SU(5)$  under the breaking  $SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$ :

$$\mathbf{5} = (\mathbf{3}, \mathbf{1})(-2) + (\mathbf{1}, \mathbf{2})(+3) , \quad (38)$$

$$\mathbf{10} = (\mathbf{1}, \mathbf{1})(+6) + (\mathbf{\bar{3}}, \mathbf{1})(-4) + (\mathbf{3}, \mathbf{2})(+1) . \quad (39)$$

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<sup>12</sup>The simplest example of an *s*-confining theory is  $SU(N_c)$  with  $N_f = N_c + 1$  flavors. In this theory we have confinement without chiral symmetry breaking at the origin of the meson and baryon moduli space.

Now let us turn on non-zero vacuum expectation values for  $(\mathbf{1}, \mathbf{1})(+6)$  in all three  $\Phi_\alpha$ , and also for  $(\mathbf{\bar{3}}, \mathbf{1})(-4)$  in at least one of the  $\Phi_\alpha$ . This is consistent with the flat directions  $\Sigma_{\{\alpha\beta\}}$ . The gauge group is broken down to  $SU(2)$ , and the left-over charged matter consists of three  $\mathbf{2}$ 's coming from  $\Phi_\alpha$  as well as three  $\mathbf{2}$ 's coming from  $Q_\alpha$ . That is, we have  $SU(2)$  with 3 flavors, which is  $s$ -confining.

Let us now include the tree-level superpotential (19). Then two of the three  $\mathbf{2}$ 's coming from  $Q_\alpha$  acquire masses, and we have  $SU(2)$  with 2 flavors. In this theory we have quantum modification of the moduli space [28], but supersymmetry is unbroken<sup>13</sup>. Finally, note that the anomalous  $U(1)_A$  D-term can also be canceled in this model.

### *The $SU(6) \otimes SO(2) \otimes U(1)_A$ Model*

This is the  $\eta = -1$  and  $N = 6$  model. The gauge group is  $SU(6) \otimes SO(2) \otimes U(1)_A$ , the chiral matter is given by  $\Phi_\alpha = 3 \times (\mathbf{15}, \mathbf{1})(+2)$  and  $Q_\alpha = 3 \times (\mathbf{\bar{6}}, \mathbf{2})(-1)$ , and the tree-level superpotential is given by (19). (Note that  $SO(2) \sim U(1)$ , and the doublet  $\mathbf{2}$  of  $SO(2)$  refers to the states with opposite  $U(1)$  charges.) The  $U(1)_A$  D-term is given by

$$D = 2\Phi^2 - Q^2 + \xi_{\text{FI}} , \quad (40)$$

where  $\xi_{\text{FI}}$  is negative for positive values of  $\text{Im}(\phi_1)$ . Note that

$$\Sigma \equiv \epsilon_{\alpha\beta\gamma} \Phi_\alpha \Phi_\beta \Phi_\gamma = (\mathbf{1}, \mathbf{1})(+6) . \quad (41)$$

Thus,  $\Sigma$  is a chiral gauge invariant operator w.r.t.  $SU(6) \otimes SO(2)$ . So we have a D-flat direction, which is also F-flat, corresponding to turning on non-vanishing expectation values of  $\Phi_\alpha$  (the  $U(1)_A$  D-term can be canceled by appropriately turning on  $\text{Im}(\phi_1)$ ). It is not difficult to see that at generic points along this flat direction the  $SU(6)$  subgroup is broken down to a  $U(1)$ , so the resulting gauge group is  $U(1) \otimes SO(2) \sim U(1) \otimes U(1)$ . In the process

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<sup>13</sup>In [23] it was argued that supersymmetry is broken in this model once we include the tree-level superpotential. In particular, the non-perturbative superpotential is known for the  $SU(5)$  theory with  $3 \times \mathbf{10}$  and  $3 \times \mathbf{\bar{5}}$  and no tree-level superpotential [27]. If we now add the tree-level superpotential and write the total superpotential in terms of the  $SU(5)$  gauge invariant degrees of freedom, we will find that this total superpotential has a runaway behavior in terms of the gauge invariant degrees of freedom. This, however, does not necessarily imply that supersymmetry is broken. Thus, for vacuum expectation values of gauge invariant operators larger than the dynamically generated scale  $\Lambda$  in the original  $SU(5)$  theory the description in terms of the  $SU(5)$  gauge invariant operators is no longer valid. Now, if supersymmetry were broken, the corresponding non-supersymmetric vacuum would have various vacuum expectation values stabilized at  $\sim M_s$  (as this stabilization is due to the contributions coming from the Kähler potential as we discussed at the end of the previous section), while in the case of a weakly coupled background ( $g_s \ll 1$ ) we have  $\Lambda \ll M_s$ . In this case we must therefore consider the low energy theory after Higgsing (and not the other way around) as we did above where we saw that supersymmetry is intact. However, if  $g_s$  is somewhat large (the one-loop  $\beta$ -function coefficient for the  $SU(5)$  theory is 9), then we could have supersymmetry breaking along the lines of [23].

of this Higgsing two of the original three  $Q_\alpha$  fields become massive, and we have total of 12 chiral supermultiplets charged under this  $U(1) \otimes U(1)$ . These supermultiplets can be used to Higgs the remaining Abelian gauge group completely.

The above discussion suggests that there is no dynamically generated superpotential in this model. Another way of arriving at the same conclusion is as follows. Consider giving vacuum expectation values to the fields  $\Phi_\alpha$  so that  $\Phi_1$  breaks  $SU(6)$  down to  $Sp(6)$ ,  $\Phi_2$  breaks  $Sp(6)$  down to  $Sp(4) \otimes Sp(2)$ , and finally  $\Phi_3$  breaks  $Sp(4) \otimes Sp(2)$  down to  $Sp(2) \otimes Sp(2) \otimes Sp(2)$  (this Higgsing is consistent with the flat direction  $\Sigma$ ). The resulting gauge group is  $SU(2) \otimes SU(2) \otimes SU(2) \otimes SO(2)$  (the anomalous  $U(1)_A$  is not shown), and the charged chiral matter is given by  $(\mathbf{2}, \mathbf{2}, \mathbf{1}, \mathbf{1})$ ,  $(\mathbf{2}, \mathbf{1}, \mathbf{2}, \mathbf{1})$ ,  $(\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1})$ ,  $(\mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{2})$ ,  $(\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{2})$ ,  $(\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2})$ . We can further break the gauge group down to  $SU(2) \otimes U(1)$  by giving non-vanishing expectation values to the last two fields. The resulting matter is given by  $\chi_i^\pm = \mathbf{2}(\pm q_i)$ ,  $i = 1, 2, 3$ , where  $\pm q_i$  are the  $U(1)$  charges. As far as the  $SU(2)$  subgroup is concerned, we have  $SU(2)$  with three flavors of quarks in the fundamental of  $SU(2)$ . In this theory we have confinement without chiral symmetry breaking (at the origin of the moduli space), and there is no non-perturbative superpotential [28]. This suggests that the  $SU(6) \otimes SO(2)$  theory with the chiral matter  $\Phi_\alpha$  and  $Q_\alpha$  and the tree-level superpotential (19) is an  $s$ -confining theory. (Such theories *without* a tree-level superpotential were classified in [27].)

### The $SU(N) \otimes SO(N-4) \otimes U(1)_A$ Models with $N \geq 7$

These are the  $\eta = -1$  and  $N \geq 7$  models. The  $SO(N-4)$  gauge invariant operators are given by mesons

$$\mathcal{M}_{[\alpha\beta]} = 3 \times (\overline{\mathbf{A}}, \mathbf{1})(-2) , \quad (42)$$

$$\mathcal{M}_{\{\alpha\beta\}} = 3 \times (\overline{\mathbf{S}}, \mathbf{1})(-2) , \quad (43)$$

and baryons

$$(\mathcal{B}_{\alpha_1 \dots \alpha_{N-4}})_{A_1 \dots A_{N-4}} = Q_{\alpha_1 A_1 i_1} \cdots Q_{\alpha_{N-4} A_{N-4} i_{N-4}} \epsilon_{i_1 \dots i_{N-4}} , \quad (44)$$

where  $A_m$  is the  $\overline{\mathbf{N}}$  index, while  $i_m$  is the  $\mathbf{N}-4$  index. (Note that the  $U(1)_A$  charge of the baryon operators is  $-(N-4)$ .) There are no  $SU(N)$  gauge invariant operators involving  $\mathcal{M}_{\{\alpha\beta\}}$ . So all the  $SU(N)$  gauge invariant operators must be constructed from  $\Phi_\alpha$ ,  $\Theta_\alpha \equiv \epsilon_{\alpha\beta\gamma} \mathcal{M}_{[\beta\gamma]}$  and the baryons  $\mathcal{B}_{\alpha_1 \dots \alpha_{N-4}}$ . If  $N$  is odd we have no gauge invariant operators involving totally antisymmetrized products of only  $\Phi_\alpha$  or only  $\Theta_\alpha$ . On the other hand, if  $N$  is even we have no such operators as  $N/2 > 3$  (and the index  $\alpha$  takes only three values). So the building blocks for the gauge invariant operators must be  $\Phi_\alpha \Theta_\beta$  and  $\Phi_{\alpha_1} \cdots \Phi_{\alpha_n} \mathcal{B}_{\beta_1 \dots \beta_{N-4}}$ , where  $n = (N-4)/2$  if  $N$  is even, and  $n = N-2$  if  $N$  is odd. This implies that if  $N$  is even all gauge invariant operators have zero  $U(1)_A$  charge. On the other hand, if  $N$  is odd the gauge invariant operators must have  $U(1)_A$  charges which are positive multiples of  $N$ .

Now, if  $\Lambda$  is the dynamically generated scale of the  $SU(N)$  theory, then  $\Lambda^{\beta_0}$  ( $\beta_0 = 3N - \frac{1}{2} \times 3 \times (N-2) - \frac{1}{2} \times 3 \times (N-4) = 9$  is the one-loop  $\beta$ -function coefficient for  $SU(N)$ ) has the  $U(1)_A$  charge



$$q_A = (+2) \times \frac{N(N-1)}{2} + (-1) \times N \times (N-4) = 3N . \quad (45)$$

This together with the above arguments indicates that in theories with even  $N$  we cannot have a non-perturbative superpotential (as the superpotential must have vanishing  $U(1)_A$  charge). For odd  $N$ , however, this argument does not rule out a non-perturbative superpotential.

To understand the odd  $N$  cases in more detail, let us use the standard  $U(1)_R$  symmetry arguments. We will assign  $+1$   $U(1)_R$  charges to the gauginos of the  $SU(N)$  as well as  $SO(N-4)$  gauge groups, and  $U(1)_R$  charges  $q_\Phi$  and  $q_Q$  to the fields  $\Phi_\alpha$  respectively  $Q_\alpha$ . Then the requirement that the  $U(1)_R SU(N)^2$  and  $U(1)_R SO(N-4)^2$  anomalies vanish gives the following values:

$$q_\Phi = \frac{2N-12}{3N} , \quad (46)$$

$$q_Q = \frac{2N+6}{3N} . \quad (47)$$

This implies that the  $(U(1)_A, U(1)_R)$  charges read:

$$\Lambda^3 : (N, 0) , \quad (48)$$

$$\Phi_\alpha \Theta_\beta : (0, 2) , \quad (49)$$

$$\Phi_{\alpha_1} \cdots \Phi_{\alpha_{N-2}} \mathcal{B}_{\beta_1 \dots \beta_{N-4}} : (N, 2(2N-9)/3) . \quad (50)$$

The superpotential must have  $U(1)_R$  charge  $+2$ . We do have combinations with  $U(1)_R$  charge  $+2$  (and  $U(1)_A$  charge 0), which can schematically be written as

$$\left( \frac{\Phi_{\alpha_1} \cdots \Phi_{\alpha_{N-2}} \mathcal{B}_{\beta_1 \dots \beta_{N-4}}}{\Lambda^3} \right)^{\frac{3}{2N-9}} = \left( \frac{[\Phi_{\alpha_1} \cdots \Phi_{\alpha_{N-2}} \mathcal{B}_{\beta_1 \dots \beta_{N-4}}]^3}{\Lambda^9} \right)^{\frac{1}{2N-9}} . \quad (51)$$

These combinations, however, are not holomorphic in the gauge invariant operators for odd  $N \geq 7$  (in particular, we have a branch point at the origin). We therefore conclude that for these values of  $N$  we do not have a non-perturbative superpotential either.

Next, recall that the tree-level superpotential is given by

$$\mathcal{W}_{\text{tree}} = y \Phi_\alpha \Theta_\alpha . \quad (52)$$

The F-flatness conditions then imply that  $\Theta_\alpha = 0$  and  $\epsilon_{\alpha\beta\gamma} \Phi_{\beta[A_1 A_2]} Q_{\gamma A_2 i} = 0$ . In particular, in all cases at hand the gauge invariant operators involving the  $SO(N-4)$  mesons  $\Theta_\alpha$  must vanish.

On the other hand, in the odd  $N$  cases the gauge invariant operators involving the  $SO(N-4)$  baryons can be written as

$$(\Phi_{\alpha_1} \Phi_{\alpha_2}) \left( (\Phi_{\beta_1} Q_{\gamma_1 i_1}) \cdots (\Phi_{\beta_{N-4}} Q_{\gamma_{N-4} i_{N-4}}) \right) \epsilon_{i_1 \dots i_{N-4}} . \quad (53)$$

Due to the aforementioned F-flatness conditions the  $\beta_m$  and  $\gamma_m$  indices must be symmetrized pairwise. Each such symmetrization gives 6 of the  $SO(3)$  global symmetry (note that 6 =

5 + 1). So we have  $(N - 4)$  6's of  $SO(3)$  completely antisymmetrized. However, at most 3 6's of  $SO(3)$  can be completely antisymmetrized without vanishing. This implies that for  $N > 7$  the above gauge invariant operators should vanish to be compatible with the F-flatness conditions, and the blow-up mode is frozen at its vanishing value. So in the odd  $N > 7$  cases the  $SO(N - 4)$  gauge subgroup is unbroken, and therefore so is the  $SU(N)$  gauge subgroup.

A similar analysis can be performed in the even  $N$  cases. Here the gauge invariant operators involving the  $SO(N - 4)$  baryons can be written as

$$\left(Q_{\alpha_1 i_1} \cdots Q_{\alpha_{(N-4)/2} i_{(N-4)/2}}\right) \left((\Phi_{\beta_1} Q_{\gamma_1 j_1}) \cdots (\Phi_{\beta_{(N-4)/2}} Q_{\gamma_{(N-4)/2} j_{(N-4)/2}})\right) \epsilon_{i_1 \dots i_{(N-4)/2} j_1 \dots j_{(N-4)/2}} \cdot \quad (54)$$

In this case we therefore have  $(N - 4)/2$  6's of  $SO(3)$  completely antisymmetrized. This implies that for  $N > 10$  these gauge invariant operators must vanish to be compatible with the F-flatness conditions, and the blow-up mode is frozen at its vanishing value. So in the even  $N > 10$  cases the  $SO(N - 4)$  gauge subgroup is unbroken, and therefore so is the  $SU(N)$  gauge subgroup.

Now, in the  $N = 7$  and  $N = 10$  cases we have 3 6's of  $SO(3)$  completely antisymmetrized. Since 6 is reducible ( $6 = 5 + 1$ ), and a totally antisymmetric product of 3 5's vanishes, we have  $1 \cdot 5 \cdot 5$  with the two 5's antisymmetrized. In particular, the singlet  $\Phi_\alpha Q_\alpha$  must be non-vanishing, that is, for at least one value of  $\alpha$  we must have  $\Phi_\alpha \neq 0$  and  $Q_\alpha \neq 0$ . Without loss of generality we can choose  $\Phi_1 \neq 0$  and  $Q_1 \neq 0$ . The F-flatness conditions then imply that either  $\Phi_2 = Q_2 = 0$  or  $\Phi_3 = Q_3 = 0$ . But then the gauge invariant operators containing  $SO(N - 4)$  baryons also vanish in the  $N = 7$  and  $N = 10$  cases.

Finally, consider the  $N = 8$  case, where we have 2 6's of  $SO(3)$  antisymmetrized. In this case the relevant products are  $1 \cdot 5$  and  $5 \cdot 5$  (the latter is antisymmetrized). So in this case the above argument does not apply. However, for  $N = 8$  we have  $SO(N - 4) = SO(4) \sim SU(2)_L \otimes SU(2)_R$ , and the matter is

$$\Phi_\alpha = 3 \times (\mathbf{28}, \mathbf{1}, \mathbf{1})(+2) , \quad (55)$$

$$L_\alpha = 3 \times (\overline{\mathbf{8}}, \mathbf{2}, \mathbf{1})(-1) , \quad (56)$$

$$R_\alpha = 3 \times (\overline{\mathbf{8}}, \mathbf{1}, \mathbf{2})(-1) . \quad (57)$$

The basic  $SU(2)_L \otimes SU(2)_R$  gauge invariant operators are

$$\mathcal{L}_\alpha = \epsilon_{\alpha\beta\gamma} L_\beta L_\gamma = 3 \times (\overline{\mathbf{28}}, \mathbf{1}, \mathbf{1})(-2) , \quad (58)$$

$$\mathcal{L}_{\{\alpha\beta\}} = L_{\{\alpha} L_{\beta\}} = 6 \times (\overline{\mathbf{36}}, \mathbf{1}, \mathbf{1})(-2) , \quad (59)$$

$$\mathcal{R}_\alpha = \epsilon_{\alpha\beta\gamma} R_\beta R_\gamma = 3 \times (\overline{\mathbf{28}}, \mathbf{1}, \mathbf{1})(-2) , \quad (60)$$

$$\mathcal{R}_{\{\alpha\beta\}} = R_{\{\alpha} R_{\beta\}} = 6 \times (\overline{\mathbf{36}}, \mathbf{1}, \mathbf{1})(-2) \quad (61)$$

The mesons  $\mathcal{L}_{\{\alpha\beta\}}$  and  $\mathcal{R}_{\{\alpha\beta\}}$  cannot enter the  $SU(8)$  gauge invariant operators, while the mesons  $\mathcal{L}_\alpha = \mathcal{R}_\alpha = 0$  due to the F-flatness conditions.

The above arguments indicate that in the  $N \geq 7$  models the gauge group is unbroken, and the blow-up mode is zero (so that the  $U(1)_A$  D-term vanishes). At low energies the theory flows into an interacting fixed point. As was pointed out in [8], in the large  $N$  limit

this superconformal field theory is actually the same as the parent  $\mathcal{N} = 4$  supersymmetric  $SO(3N - 4)$  gauge theory (which, in turn, in the large  $N$  limit is the same as the parent  $\mathcal{N} = 4$  supersymmetric  $SU(3N - 4)$  gauge theory).

## V. OTHER EXAMPLES

In this section we would like to briefly mention other interesting  $\mathbf{Z}_p$  examples. In particular, we will discuss a  $\mathbf{Z}_5$  example and a  $\mathbf{Z}_7$  example.

### A. A $\mathbf{Z}_5$ Example

Consider the  $\mathbf{Z}_5$  orbifold group whose generator  $\theta$  has the following action on the complex coordinates  $z_\alpha$  ( $\omega \equiv \exp(2\pi i/5)$ ):

$$\theta z_{1,2} = \omega z_{1,2} , \quad \theta z_3 = \omega^3 z_3 . \quad (62)$$

The tadpole cancellation conditions have the following solution (see subsection B of section III):

$$\text{Tr}(\gamma_{\theta^2}) = -4\eta \frac{1}{(1+\omega)^2(1+\omega^3)} = -4\eta(\omega + \omega^4) . \quad (63)$$

This then implies that

$$\gamma_\theta = \text{diag}(I_N, \omega I_N, \omega^2 I_{N-4\eta}, \omega^3 I_{N-4\eta}, \omega^4 I_N) . \quad (64)$$

In particular, consider the  $\eta = -1$  and  $N = 0$  case. We then have  $SU(4) \otimes U(1)_A$  gauge group with a chiral supermultiplet in  $\mathbf{6}(+2)$ . As far as the non-Abelian part of the gauge group is concerned, we can view this model as  $SO(6)$  with one vector. In this theory we have a runaway superpotential [24], so supersymmetry is broken in this model.

### B. A $\mathbf{Z}_7$ Example

Consider the  $\mathbf{Z}_7$  orbifold group whose generator  $\theta$  has the following action on the complex coordinates  $z_\alpha$  ( $\omega \equiv \exp(2\pi i/7)$ ):

$$\theta z_1 = \omega z_1 , \quad \theta z_2 = \omega^2 z_2 \quad \theta z_3 = \omega^4 z_3 . \quad (65)$$

The tadpole cancellation conditions have the following solution (see subsection B of section III):

$$\gamma_\theta = -4\eta . \quad (66)$$

This then implies that

$$\gamma_\theta = \text{diag}(I_{N-4\eta}, \omega I_N, \omega^2 I_N, \omega^3 I_N, \omega^4 I_N, \omega^5 I_N, \omega^6 I_N) . \quad (67)$$

In particular, consider the  $\eta = -1$  and  $N = 0$  case. We then have pure  $SO(4) \sim SU(2)_L \otimes SU(2)_R$  super-Yang-Mills theory. It would be interesting to use this setup to identify BPS domain walls in this gauge theory.

## VI. CONCLUDING REMARKS

The above discussions illustrate that the orientiworld framework has a rich variety of non-perturbative phenomena that can arise in the orientiworld models. In particular, we can have dynamical supersymmetry breaking as well as various interesting supersymmetry preserving phenomena such as confinement, domain walls, *etc.*

The orientiworld framework gives a consistent embedding of non-conformal gauge theories in the Type IIB string theory context. Generalizing the gauge/string theory correspondence of [29–31] to such theories would be very interesting, but it is also expected to be non-trivial as the corresponding supergravity solutions are often expected to be singular. Solving the problem of such singularities might also shed light on non-supersymmetric cases, which would be interesting in the context of the cosmological constant problem along the lines of [32].

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